ICM - 82
Section 16

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ENUMERATION OF ACYCLIC SUPERTOURNAMENTS OF A FINITE LABELED ACYCLIC DIGRAPH.

Let G denote a finite, labeled, acyclic digraph. We define a tournament a complete oriented graph. A tournament which is a spanning subgraph of a given digraph is called a supertournament of this digraph. Let $t(G)$ be the number of all different acyclic supertournaments of G.

A recurrent method of canceling cycles of a digraph is presented. Using this method we show

Theorem 1. For any finite, labeled, acyclic digraph G the problem of calculating $t(G)$ is reducible to the case of finite, labeled, oriented rooted trees.

Let $V(G)$ be the set of vertices of a digraph G and let T denote a finite, labeled, oriented rooted tree. $\forall x \in V(T)$ let T_x be the maximal, oriented, rooted subtree with the root x in T.

Theorem 2.
$$t(T) = |V(T)|! \Big/ \prod_{x\in V(T)} |V(T_x)|$$

Furthermore, two families of upper bounds are deduced.

Lemma. $(G_* \subseteq G \subseteq G^*) \wedge (V(G_*) = V(G) = V(G^*)) \Longrightarrow t(G^*) \leqslant t(G) \leqslant t(G_*)$.

Let $T/G/$ denote an oriented, rooted spanning tree of a weakly connected digraph G and let $I_n \hat{=} \{1,2,\ldots,n\}$.

Theorem 3. $\forall\, T/G/$:
$$t(G) \leqslant |V(G)|! \Big/ \prod_{x\in V(G)} |V(T_x/G/)|$$

A minimal Dilworth decomposition of G is defined as follows:
$$V(G) = \bigcup_{i=1}^{q(G)} V(C_i); \quad \forall\, i\neq j:\ V(C_i) \cap V(C_j) = \emptyset,$$
where the C_i are paths of the transitive closure $\bar{G}$ and $q(G)$ is the maximal defiency of G.

Theorem 4. Let G be a finite, labeled, acyclic, weakly connected digraph with exactly one initial vertex $b \in V(G)$. Then, for any minimal Dilworth decomposition of G we have
$$t(G) \leqslant \frac{(|V(G)| - 1)!}{(|V(C_p)| - 1)!} \prod_{\substack{i=1\\ i\neq p}}^{q(G)} |V(C_i)|!$$
where $b \in V(C_p)$, $p \in I_{q(G)}$.