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ENUMERATION OF ACYCLIC SUPERTOURNAMENTS
OF A FINITE LABELED ACYCLIC DIGRAPH.

Let $G$ denote a finite, labeled, acyclic digraph. We define a
tournament a complete oriented graph. A tournament which is a span-
ning subgraph of a given digraph is called a supertournament of this
digraph. Let $t(G)$ be the number of all different acyclic supertour-
naments of $G$.

A recurrent method of canceling cycles of a digraph is presen-
ted. Using this method we show

**Theorem 1.** For any finite, labeled, acyclic digraph $G$ the problem of
calculating $t(G)$ is reducible to the case of finite, labeled, ori-
ented rooted trees.

Let $V(G)$ be the set of vertices of a digraph $G$ and let $T$ denote
a finite, labeled, oriented rooted tree. For $x \in V(T)$ let $T_x$ be the
maximal, oriented, rooted subtree with the root $x$ in $T$.

**Theorem 2.**

$$t(T) = \frac{|V(T)|}{\prod_{x \in V(T)} |V(T_x)|}$$

Furthermore, two families of upper bounds are deduced.

**Lemma.** ($G_x \subseteq G \subseteq G^x$) $\land (V(G_x) = V(G) = V(G^x))$ $\implies t(G^x) \leq t(G) \leq t(G_x)$.

Let $T/G$ denote an oriented, rooted spanning tree of a weakly
connected digraph $G$ and let $I_n = \{1, 2, \ldots, n\}$.

**Theorem 3.**

$$V(T/G): \quad t(G) \leq \frac{|V(G)|}{\prod_{x \in V(G)} |V(T_x/G)|}$$

A minimal Dilworth decomposition of $G$ is defined as follows:

$$q(G) = \bigcup_{i=1} V(C_i); \quad V i\neq j: V(C_i) \cap V(C_j) = \emptyset,$$

where the $C_i$ are paths of the transitive closure $\bar{G}$ and $q(G)$ is the
maximal deficieny of $G$.

**Theorem 4.** Let $G$ be a finite, labeled, acyclic, weakly connected
digraph with exactly one initial vertex $b \in V(G)$. Then, for any mini-
mal Dilworth decomposition of $G$ we have

$$t(G) \leq \frac{(|V(G)| - 1)!}{(\prod_{i=1}^{q(G)} |V(C_i)|)!} \prod_{i \neq p}^{q(G)} |V(C_p)|$$

where $b \in V(C_p), p \in I_{q(G)}$. 