OPTIMIZATION OF CONNECTION STRUCTURES IN GRAPHS

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Program System NETOPT

Abstract

NETOPT is a system of software tools for designing layouts of networks in two- or three-dimensional spaces with intense inherent structure. For each concrete project the layout possibilities are modelled by a directed or undirected graph embedded in the generally threedimensional spatial region in which the network has to be laid out. NETOPT supports the input and modification of such graphs, serves for the optimization of certain connection structures in them, performs the combination of such connection structures to entire network layouts in the case of cable networks, and presents intermediate and final results with a degree of aggregation that can be chosen by the user. Thus, the program system includes not only parts for the solution of general optimization problems in graphs but also a variety of possibilities to consider special technological and commercial aspects in the field of cable network layout. The entire program system is characterized by an extensive support of man-machine interaction. All parts of NETOPT are written in FORTRAN 77. Presently the system is running on 32-Bit computers of the type CM 52.

1. General Characterization of NETOPT

Many practical design problems regard the optimal layout of networks in generally three-dimensional spaces with intense inherent structures: Energy supply or decentralized automation systems have to be laid out in environments that are far from being a "greenfield site" and the optimization of connection structures for transportation purposes very often regards spaces in which traffic systems already exist.

NETOPT is a program system developed for the solution of layout design problems of this kind. For each concrete project the layout possibilities are modelled by a directed or undirected graph embedded in the space in which the network has to be laid out. The edges (or arcs) of the graph (or digraph) represent routing possibilities. Its nodes stand for crossings and branchings of such routing possibilities, for a priori fixed connections of the network to other systems, and for possibilities of the placement of network components as power distributors or supervisory stations.

Both the nodes and edges (arcs) can be characterized with respect to the concrete application. For instance, the weights assigned to the edges (arcs) may represent

- lengths of cable line sections,
- costs for the construction of such sections along the routes represented by the corresponding edges,
- costs for the modernization of a railway line, or
- running-times of cars along street segments.

Often it is reasonable to assign to each edge (arc) a length and additionally a certain code characterizing the type of the link represented by the edge (arc). For instance, in the case of cable system layout for each edge this code specifies the type of the cable line section required if a line is really constructed along the routing possibility represented by the corresponding edge.

The nodes of the graph (digraph) in which the placement of system components is admissible may be characterized by weights standing for the cost of component installation and/or measures of protection.

NETOPT supports the input and modification of graphs or digraphs as well as the checking of certain structural properties.

This regards not only the modelling of layout possibilities in the case of real practical design projects but also the fast generation of fictive models utilizable for efficiency analysis with respect to graph algorithms, for instance. Within NETOPT this generation is carried out ba a random graph generator /1/ so that the system user has to specify only very few parameters.

In the case of those practical applications for which NETOPT has been mainly developed, the graphs (or digraphs) describing routing and placement possibilities are sparse. Therefore, both the real, project-oriented and the fictive graph models are stored in the form of certain neighbour lists that are efficient with respect to memory requirements as well as information access. For both undirected and directed graphs the kind of internal representation is the same: Undirected graphs are represented as directed ones with two arcs in opposite direction between each two vertices.

In the case of cable system layout the concrete project must be described besides the graph model by further project-specific, general technological and commercial information. The project-specific data regard (s. also /2/)

- the admissible basic structure(s) of the entire cable network,
- characteristics of customers (for instance their power consumption), and
- further technological information with respect to restrictions and requirements regarding the entire project.

General technological and commercial information being not dependent on the specific cable project includes

- characterisitcs of all types and variants of cable lines,
- characteristics of parts and accessories for the construction of lines for cable routing,

- characteristics of different cable types (especially prices),
- the cable cross sections in dependence on power consump tions of the customers, necessary cable lengths and other factors (according to corresponding special regulations).

The main part of NETOPT is a system of implemented algorithms for the exact or approximative determination of certain cost-minimal connection structures within graph models and - in the case of cable network layout - for the combination of such structures as subproblem solutions to the entire layout. This optimization part of NETOPT consists of three subsystems:

- NETLAY — regards mainly the optimization of noninterfering paths and trees and the subsystem combination,
- THERMLAY — concerns the optimization of systems of interacting paths and of Steiner trees in graphs on the basis of thermodynamically motivated methods,
- DISPATH — serves for the optimization of node-disjoint multiple paths in graphs.

These three software subsystem use the same external data structure for the description of layout problems. This regards graphs or digraphs representing layout possibilities as well as the above mentioned additional technological and commercial data bases. A further link between the subsystems consists in the possibility to exchange solutions of optimization problems. For instance, a Steiner tree representing a cost-effective system of cable lines within an industrial site can be determined by THERMLAY and using this result NETLAY may calculate an optimal component placement within the tree.

According to the complexity of real practical layout design problems (especially in the case of cable system layout) the entire program system is characterized by an extensive support of man-machine interaction. This regards both the modelling and the optimization process, i. e. the input of project data, the generation of fictive graph models for the performance analysis of algorithms, the decomposition of layout projects into subproblems, the choice of corresponding solution methods and the selection of determined connection structures for the formation of complete network layouts. All parts of NETOPT are written in FORTRAN 77. Presently the system is running on a 32-Bit computer of the type CM52.

The first extensive application of NETOPT was in the field of cable network layout in an industrial site. The graph representing the layout possibilities contained 644 nodes and 814 edges. In this case the usage of NETOPT led to a layout solution for which the total costs are by 27 % smaller than those of a solution found by design engineers on the base of conventional methods /2/.
Fig. 1 shows the simplified structure of the entire program system.

2. Subsystem NETLAY

As already partially mentioned NETLAY serves for

- the optimization of noninterfering paths, the calculation of corresponding distances, and the determination of optimal placements for the centers of star networks in digraphs,
- the determination of Minimum Spanning and Steiner trees in graphs,
- technological and commercial calculations with respect to cable systems, and
- in the case of cable network layout the combination of subsystems into complete networks.

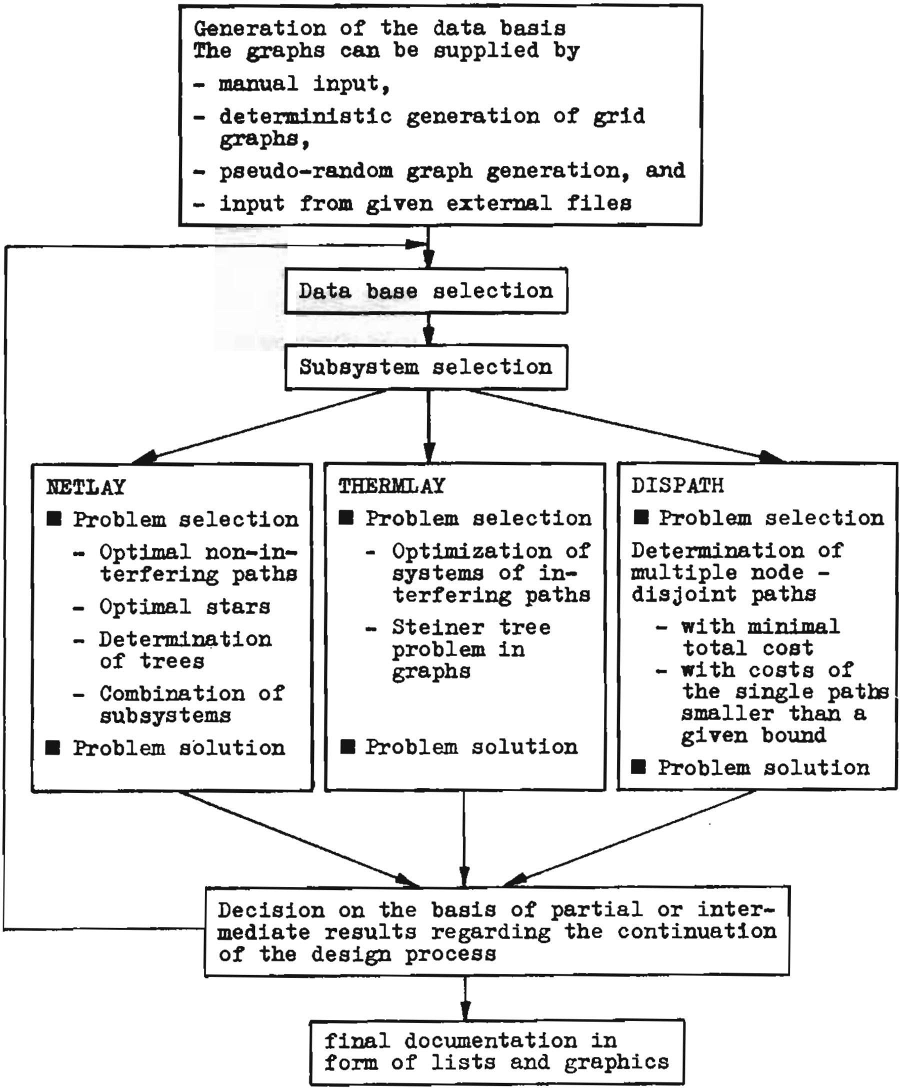

Fig. 1: Structure of program system NETOPT

All implemented optimization algorithms are deterministic. With the exception of the Steiner Tree procedures they are all correct, i. e. they yield an exact solution of the corresponding problem. Steiner Trees are determined by a heuristic approach that approximately solves the NP-complete problem in polynomial time. This heuristic is "good" in the sense that it guarantees

$$C_H \leq 2(1 - \frac{1}{M})C_{opt}, \qquad (1)$$

where C_H and C_{opt} are the costs of the approximate and exact solution, respectively, and M denotes the number of vertices that must be connected by the tree /3/.

In the following we list the different functions of NETLAY and refer to the underlying algorithms.

Optimal Paths

One complex of routines regards the determination of optimal paths and distances in digraphs by new algorithms based on the methods of Dijkstra, Dreyfus and Yen (the corresponding mathematical problem formulations, algorithms and their computational complexity are presented in /4/):

- Determination of optimal ("shortest") paths and the corresponding costs
  - from a speoified vertex to all other vertices (Problem 1 in /4/),
  - from one specified vertex to another specified vertex (Problem 2 in /4/),
  - from a specified vertex to all other vertices of a given vertex subset (Problem 3 in /4/),
  - from each vertex to each other vertex of the digraph (Problem 6 in /4/),
  - from each vertex of a given subset to each other vertex of this subset (Problem 7 in /4/),

  - from each vertex of a vertex subset X to each vertex of a vertex subset Y (Problem 8 in /4/).

- Determination of minimal distances
  - from a specified vertex to a vertex subset (Problem 4 in /4/),
  - from a vertex subset X to a subset Y (Problem 5 in /4/).

Optimal Layout of Star Networks

Another complex regards the optimal layout of star networks in graphs modelling the layout possibilities. This optimization problem includes both optimal placement of the star center and optimal routing of the single links. Mathematical problem formulations, algorithms and computational complexity are presented in /5/. Of course the solution methods are based on path optimization. The complex consists of the following three functions:

- Solution of the 1-median problem, i. e. determination of that vertex of a given connected graph for which the sum of distances (costs of optimal paths) to all other vertices of the graph is smallest.
- Solution of the generalized 1-median Problem /5/, i. e. determination of that vertex v^* out of a given vertex subset X for which the sum of distances to all elements of another given vertex subset increased by a "placement weight" assigned to v^* is smallest (such weights are assigned to all elements of X and may represent the costs for the installation of a network component in the corresponding location).
- Determination of the layout of star networks with preference to parallel routing.

Within the procedure for the solution of the generaozed 1-median problem paths are modified by a heuristic approach so that the paths run parallel to a great extent, if this is possible without increasing the cost of each single path.

The practical background of this design problem is cable routing in industrial sites where the total layout costs can be often decreased by a bunching of cables.

Determination of Trees

This complex refers to the determination of Minimum Spanning Trees and Steiner trees in graphs:

- Determination of a Minimum Spanning Tree of a given graph on the basis of Prim's algorithm.
- Determination of an approximate solution of the Steiner Tree Problem in graphs for which inequality (1) holds.

The basic idea used for the determination of Steiner trees is that described in /3/. However, with respect to an implementation especially efficient for the treatment of the Steiner Tree Problem in large sparse graphs a variety of new specifications was developed /6/.

Technological and Commercial Calculations with Respect to Cable Systems

This complex treats technological and commercial aspects of cable system design. It includes such basic procedures as the determination of cable cross-sections in dependence on required cable lengths and power consumptions of customers.

Combination of Subsystems into Complete Networks Layouts

In most real situations cable network layout problems are so complex that an exact optimization is absolutely impracticable. Therefore, it is reasonable to decompose a given layout project heuristically into such subproblems for which sufficiently fast solution methods are available. NETOPT offers a variety of software tools for the exact or approximative determination of subproblem solutions. For instance, it is possible to desgin at first Steiner Trees as cost effective line systems for the layout of different cable subsystems with star structure being parts of the entire cable project, and secondly to determine optimal placements of the star centers within the line systems. In any case finally the generally overlapping solutions of subproblems must be combined into the layout of the given entire network. This is carried out by the procedures of the complex under discussion. It includes the determination of the required line types on the basis of the actual occupancy of all line system sections with cables of different cross-sections. Furthermore this complex regards the calculation of costs (for cables, for the preparation of cable lines, especially for the required parts and accessories, for cable and component installations and for wages) and the generation of corresponding line occupancy and cable lists (s. also /2/).

3. Subsystem THERMLAY

THERMLAY is based on the thermodynamically motivated simulation, a modern stochastic algorithm /7/, /8/. The following two NP-complete graph optimization problems can be solved:

- Optimization of interacting path systems with the same initial vertex (star) and nonlinear cost functions in graphs. (Problem 1 in /9/)
- Steiner Tree Problem in graphs. (Problem 2 in /9/)

Furthermore it is possible to generate special basic graphs /9/ which allow a colour-graphical representation of final or intermediate solutions of the optimization problems.

Optimization of Interacting Path Systems

This problem arises, if the total cost of a star does not depend linearly on the costs of single paths, but on the sum of costs of all arcs belonging to the star and on the sum of path costs. In practicel there exists a wide variety of relationships between these both cost components and corresponding cost computations. The following two possibilities for the computation of star costs can be executed by THERMLAY:

- The computation of path costs with respect to given path types and the computation of arc costs depending on given arc types as well as on the number of paths containing this arc.
- A cost procedure for a practically relevant cable routing problem:
  - At first the type of all cables is computed. (The type of a cable is its cross-section.)
  - The computation of cable line costs depends on given cable line types and cable line capacities. The required capacities depend on the number and cross-sections of all cables lying in the cable lines. The cable costs are computed with respect to cable and cable line types.

Remark:
Other possible interacting path cost computations may be carried out by modifying the cost procedure and other required procedures (for example: computation of cross-sections). The main part of the program can be used without changes.

The following parameters can be set interactively:

- vertex number of the star root
- a strategy parameter regarding changes of the actual structure of the path system
- number of path types
- some technological parameters with respect to cable systems
- control parameter for computing the initial star for the thermodynamically motivated simulation:
  - Fast computation of a feasible initial star
  - Reading of the star solution generated by the program system NETLAY.

Then the thermodynamically motivated simulation is carried out depending on the three interactively chosen parameters cycle length, initial temperature and reduction factor /9/.

Steiner-Tree-Problem

Parameters regarding the initialization of the algorithm and technological aspects with respect to cable system layout can be set interactively. This concerns for instance the choice of the strategy for the construction of an initial tree:

- Fast computation of a feasible tree
- Reading of a solution generated with the program package NETLAY.

After that the thermodynamically motivated simulation is carried out.
During the simulation it is possible to observe the development of costs of the changing structure on the terminal. By using a colour graphical representation the changing structure can be observed on the terminal.
Furthermore a partial sequence of structures with decreasing costs can be stored on a special file. After one simulation it is possible to carry out a next simulation with new parameters.

4. Subsystem DISPATH

DISPATH serves for the determination of k node-disjoint paths between two given vertices of an undirected graph

- with minimal total cost (Problem 1 in /10/) or
- with costs of each of the single paths less than a bound B (NP-complete problem, Problem 2 in /10/).

The procedure yields an exact solution of each problem by using a depth-first strategy. It is possible to select either only node costs or only edge costs or both for the calculation of path costs. In many cases by Procdure ADMIS (s. /10/) an admissible solution can be found which yields an upper bound for the cost of the total path system and for the costs of the single paths. These bounds or an estimation of them or an a priori given bound B in the case of Problem 2 are used for both graph reduction and optimization in the reduced graph /10, 11/. The graph reduction is carried out in order to eliminate those parts of the given graph which are irrelevant to the optimization problem.

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